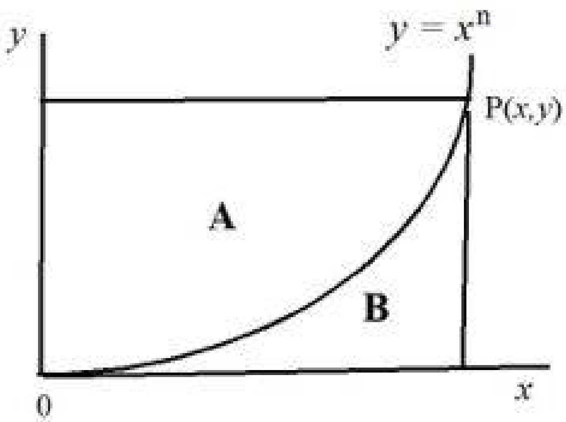
Some thoughts on the integral power rule

by Sidney Schuman (Lewisham College 1992)



In reference to this digram, here's how we know that $\frac{A}{B} = n$:

Given that $\int x^n = \frac{x^{n+1}}{n+1}$ (constant of integration omitted) then $B = \frac{x^{n+1}}{n+1}$ **Equation 1**

$A + B = xy \quad \therefore A + B = x^{n+1} \quad \therefore A = x^{n+1} - B \quad \therefore A = x^{n+1} - \frac{x^{n+1}}{n+1}$ and

$A(n+1) = (n+1)\,x^{n+1} - x^{n+1} \quad \therefore A = n\,\frac{x^{n+1}}{n+1}$ Equation 2

Comparing Equation 1 to Equation 2 we see that $\frac{A}{B} = n$

Furthermore, here's how we know that this can be used to obtain the integral power rule:

Given that $A + B = xy = x^{n+1}$ and $\frac{A}{B} = n$ then it follows that

$A = Bn \quad \therefore Bn + B = x^{n+1} \quad \therefore B(n+1) = x^{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$.

I used this as a pre-calculus taster by asking students to show that $\frac{A}{B} = n$ as a purely calculator exercise using the mid-ordinate rule with ten vertical strips, $x = 10$ and n = 2, 3, 4 and 5. They **were encouraged to see that the approximate results were due to the low number of vertical strips used. How the accuracy could be improved by using a greater number and finally an exact result could be expected with an infinite number of vertical strips. They were then encouraged to obtain the integral power rule, with some guidance from me with the algebra.**